Guided Notes

MAC 2312

Section 7.3
Trig substitutions

$\sqrt{a^2-x^2}$	$\boxed{x = a\sin\theta}$	$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$	$1-\sin^2\theta = \cos^2\theta$
$\sqrt{a^2+x^2}$	$\boxed{x = a\tan\theta}$	$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$	$1+\tan^2\theta = \sec^2\theta$
$\sqrt{x^2-a^2}$	$\boxed{x = a\sec\theta}$	$0 \le \theta \le \frac{\pi}{2}$ or $\pi \le \theta \le \frac{3\pi}{2}$	$\sec^2\theta - 1 = \tan^2\theta$

Example 1

6 $\displaystyle\int_0^3 \frac{x}{\sqrt{36-x^2}}\,dx$

use $\sqrt{a^2-x^2}$

$a = 6$

let $x = 6\sin\theta$

$dx = 6\cos\theta\,d\theta$

$$\int_{x=0}^{x=3} \frac{6\sin\theta}{\sqrt{36-36\sin^2\theta}}(6\cos\theta\,d\theta)$$

$$= \int_{x=0}^{x=3} \frac{6\sin\theta}{6\sqrt{\cos^2\theta}} \cdot 6\cos\theta\,d\theta$$

$$= \int_{x=0}^{x=3} \frac{6\sin\theta \cdot \cancel{6\cos\theta}}{\cancel{6\cos\theta}}\,d\theta = \int_{x=0}^{x=3} 6\sin\theta\,d\theta$$

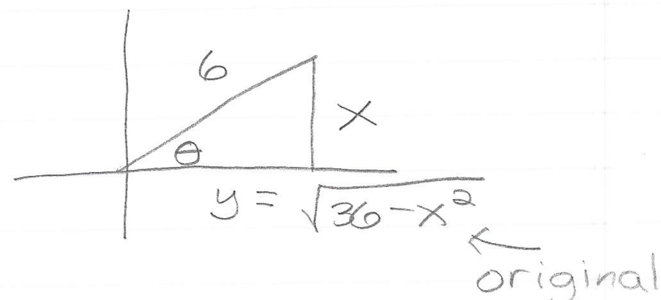
⑥ continued

$$= -6\cos\theta \Big|_{x=0}^{x=3}$$

either change the limits or change θ back to x

Recall $x = 6\sin\theta$

so $\frac{x}{6} = \sin\theta$



$$6^2 = x^2 + y^2$$
$$6^2 - x^2 = y^2$$
$$\sqrt{36 - x^2} = y$$

$$\cos\theta = \frac{adj}{hyp} = \frac{\sqrt{36-x^2}}{6}$$

$$= -\cancel{6}\left[\frac{\sqrt{36-x^2}}{\cancel{6}}\right]\Big|_{x=0}^{x=3}$$

$$= -1\left[\sqrt{36-9} - \sqrt{36-0}\right]$$

$$= -1\left(\sqrt{27} - 6\right) = 6 - \sqrt{27} = \boxed{6 - 3\sqrt{3}}$$

Are there other ways to solve this problem?

yes → <u>u sub</u>

(24) $\int_0^1 \sqrt{x - x^2}\, dx$

we need to complete the square

$-(x^2 - x)$

$-(x^2 - x + (\frac{-1}{2})^2) + (\frac{1}{2})^2$

$-(x - \frac{1}{2})^2 + \frac{1}{4}$

$\frac{1}{4} - (x - \frac{1}{2})^2$

$$\int_0^1 \sqrt{\frac{1}{4} - (x - \frac{1}{2})^2}\, dx$$

form: $\sqrt{a^2 - x^2}$ $\quad a = \frac{1}{2}$ $\quad x - \frac{1}{2} = \frac{1}{2}\sin\theta$

$dx = \frac{1}{2}\cos\theta\, d\theta$

$$= \int_0^1 \sqrt{\frac{1}{4} - (\frac{1}{2}\sin\theta)^2} \cdot \frac{1}{2}\cos\theta\, d\theta$$

when $x = 0$

$0 - \frac{1}{2} = \frac{1}{2}\sin\theta$

$-1 = \sin\theta$

$-\pi/2 = \theta$

$$= \int_0^1 \sqrt{\frac{1}{4} - \frac{1}{4}\sin^2\theta}\ \frac{1}{2}\cos\theta\, d\theta$$

$x = 1$ $\quad 1 - \frac{1}{2} = \frac{1}{2}\sin\theta$

$\frac{1}{2} = \frac{1}{2}\sin\theta$

$\theta = \pi/2 \quad 1 = \sin\theta$

$$= \int_0^1 \frac{1}{2}\sqrt{\cos^2\theta} \cdot \frac{1}{2}\cos\theta\, d\theta$$

$$= \int_0^1 \frac{1}{4}\cos^2\theta\, d\theta$$

use the identity $\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$

$$= \frac{1}{4}\int_{x=0}^{x=1} \frac{1}{2}(1 + \cos 2\theta)\, d\theta = \frac{1}{8}\left[\theta + \frac{1}{2}\sin 2\theta\ \right|_{\theta = -\pi/2}^{\theta = \pi/2}$$

$$= \frac{1}{8}\left[\frac{\pi}{2} + \overset{0}{\frac{1}{2}\sin 2(\frac{\pi}{2})} - (\frac{-\pi}{2}) - \overset{0}{\frac{1}{2}\sin 2(\frac{-\pi}{2})}\right] = \boxed{\frac{\pi}{8}}$$

7.3

30 $\displaystyle\int_{0}^{\pi/2} \frac{\cos t}{\sqrt{1+\sin^2 t}}\,dt$

$u = \sin t$
$du = \cos t\,dt$

$x = \pi/2 \quad u = \sin \pi/2 = 1$

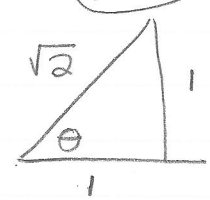
$x = 0 \quad u = \sin 0 = 0$

$$\int_{u=0}^{u=1} \frac{1}{\sqrt{1+u^2}}\,du$$

$u = \tan\theta$
$du = \sec^2\theta\,d\theta$

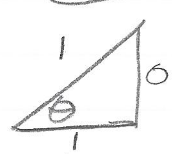
$$\int_{u=0}^{u=1} \frac{1}{\sqrt{1+\tan^2\theta}} \cdot \sec^2\theta\,d\theta$$

$u=1 \quad \tan\theta = 1$



$$\int_{u=0}^{u=1} \frac{1}{\sec\theta} \cdot \sec^2\theta\,d\theta$$

$u=0$



$\tan\theta = 0 \quad \theta = \frac{\pi}{2}$

$$\int_{u=0}^{u=1} \sec\theta\,d\theta = \ln|\sec\theta + \tan\theta| \Big|_{u=0}^{u=1}$$

$$= \ln|\sqrt{2}+1| - \ln|1+0|$$

$$= \boxed{\ln|\sqrt{2}+1|}$$